

CALCULATION OF THE D AND B MESON LIFETIMES AND THE UNITARITY TRIANGLE PARAMETERS

VICTOR CHERNYAK

Budker Institute of Nuclear Physics,
630090 Novosibirsk-90
and
Novosibirsk State University

Abstract

Using the expansions of the heavy meson decay widths in the heavy quark mass and QCD sum rules for estimates of corresponding matrix elements, we calculate the $D^{\pm,o,s}$ and $B^{\pm,o,s}$ meson lifetimes. The results for D mesons are in a reasonable agreement with the data, while it is predicted that $[\Gamma(B^o) - \Gamma(B^-)]/\Gamma_B \simeq 4\%$, and the lifetime difference of the B^o and B_s mesons is even smaller. The role of the weak annihilation and Pauli interference contributions to the lifetime differences is described in detail. In the course of self-consistent calculations the values of many parameters crucial for calculations with charmed and beauty mesons are found. In particular, the (pole) quark masses are: $M_c \simeq 1.65 \text{ GeV}$, $M_b \simeq 5.04 \text{ GeV}$, and the decay constants are: $f_D(M_c) \simeq 165 \text{ MeV}$, $f_B(M_b) \simeq 120 \text{ MeV}$. It is also shown that the nonfactorizable corrections to the $B^o - \bar{B}^o$ mixing are large, $B_B(M_b) \simeq (1 - 18\%)$. The values of the unitarity triangle parameters are found which are consistent with these results and the data available (except for the NA31 result for the ϵ'/ϵ which is too large): $|V_{cb}| \simeq 4.2 \cdot 10^{-2}$, $|V_{td}| \simeq 1.3 \cdot 10^{-2}$, $|V_{ub}/V_{cb}| \simeq 0.10$, $\{A \simeq 0.86, \rho \simeq -0.40, \eta \simeq 0.20\}$.

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1. Introduction

Experimentally measured properties of heavy mesons offer the possibility to find out values of parameters which are of fundamental importance for the Standard Model: M_c , M_b , V_{cb} , V_{ub} , V_{td} (also f_D , f_B , etc). Below the attempt is described [1] of self-consistent calculation of all the above parameters using the available experimental data on the D and B mesons. The theoretical methods used are: expansions in the heavy quark mass for obtaining the effective Lagrangians and QCD sum rules for estimates of corresponding matrix elements. The used scheme of calculations looks as follows:

Experiment $\Gamma(D \rightarrow e\nu + X) : \rightarrow$ determination of M_c ;

M_c + Heavy meson mass formulae: \rightarrow calculation of M_b ;

M_c , M_b + QCD sum rules: \rightarrow calculation of the decay constants f_D , f_B and the matrix elements $\langle M | L_{eff} | M \rangle$;

Calculation of the $D^{\pm,0,s}$ meson lifetimes and predictions for the $B^{\pm,0,s}$ meson lifetime differences and branching ratios;

Calculation of the matrix element: $\langle \bar{B}^0 | L_{eff}^{mixing} | B^0 \rangle$, prediction of $\Gamma(B_s^{short}) - \Gamma(B_s^{long})$.

Experiment $\Gamma(B \rightarrow e\nu + X) : \rightarrow$ determination of $|V_{cb}|$;

Experiment $\Delta m_d : \rightarrow$ determination of $|V_{td}|$;

Experiment on $\epsilon_K : \rightarrow$ determination of ρ , η , V_{ub} , estimates of ϵ'/ϵ , predictions for CP-violation in B-decays, etc.

2. The history

It is a long standing challenge for theory to calculate the D and B meson decay widths. On the qualitative side, two mechanisms were invoked to explain the pattern of the D meson lifetime differences: weak annihilation (WA) [2], and Pauli interference (PI) [3], [4]. As for WA, it was expected that because an admixture of the wave function component with an additional gluon or the emission of a perturbative gluon, both remove a suppression due to helicity conservation (which leads to $Br(\pi \rightarrow e\nu)/Br(\pi \rightarrow \mu\nu) \sim 10^{-4}$), the D^0 meson decay width is enhanced. On the other hand, it was expected that the destructive PI of two d-quarks (the spectator and those from a final state) will suppress the D^\pm meson decay width. As for WA, there were no reliable calculations at all. For PI, simple minded estimates (see sect.7) give too large an effect which results in a negative D^\pm decay width.

For B mesons, it was clear qualitatively that all the above effects which are of a pre-asymptotic nature and die off at $M_Q \rightarrow \infty$, will be less important. However, because the pattern of the D meson lifetime differences was not really explained and well understood, this prevented to obtain reliable estimates of the B meson lifetime differences, and only order of magnitude estimates were really available: $[\delta\Gamma(B)/\Gamma_B] : [\delta\Gamma(D)/\Gamma_D] \sim O(f_B^2 M_c^2 / f_D^2 M_B^2) \sim O(10^{-1})$.

Moreover, as for WA contributions through perturbative gluon emission (which is formally a leading correction $\sim O(\Lambda_{QCD}/M_Q)$ to the decay width and was expected before to be potentially the most important), it has been emphasized recently [5] that such contributions are of no help at all because, being large (at least formally at $M_Q \rightarrow \infty$) term by term, they cancel completely in the inclusive widths, both $O(1/M_Q)$ and $O(1/M_Q^2)$ terms. It will be shown below (see sects.8-10) that, nevertheless, there are important WA contributions but on the nonperturbative level.

Considerable progress has been achieved recently in applications of the operator product expansion to the calculation of the heavy meson decay width. In particular, it was shown that there are no $O(\Lambda_{QCD}/M_Q)$ corrections to the Born term and first nonperturbative corrections $O(\Lambda_{QCD}^2/M_Q^2)$ were calculated explicitly [6], [7]. However, these contributions are all nonvalence and so have nothing to do with lifetime differences. They are important however for the calculation of the absolute decay rates. There appeared a number of papers where these results (neglecting the four-fermion operator contributions) were applied to determine the values of the quark masses, M_c and M_b , and $|V_{cb}|$ [8], [9], [10]. However, because the four-fermion operator contributions are of crucial importance for the D mesons and lead to $\tau(D^+)/\tau(D^0) \simeq 2.5$, the real accuracy of the above results remained unclear.¹

3. The heavy quark masses

As most of calculations with heavy quarks are highly sensitive to precise values of their masses, it is of prime importance to know these as precisely as possible.

Below the pole masses, M_c and M_b , are used for the charm and bottom quarks as most convenient gauge and renormalization scheme independent quantities. Besides, the most convenient expansion parameter of the Heavy Quark Effective Theory (HQET)[12] is just the pole quark mass. On the other hand, because the pole mass receives contributions from the infrared region, the perturbative series connecting it with the current mass (say, \overline{MS} -mass) is divergent due to renormalon effects [13]:

$$M_Q = \overline{M}_Q \left\{ 1 + \frac{4}{3} \frac{\alpha_s(M_Q)}{\pi} + 1.56b_o \left(\frac{\alpha_s(M_Q)}{\pi} \right)^2 + \dots \right\} \equiv \overline{M}_Q \left\{ 1 + \frac{4}{3} \frac{\alpha_s(M_Q)}{\pi} \kappa_Q^{(m)} \right\} \quad (1)$$

Because the series in Eq.(1) is divergent, the result contains an ambiguity of order $O(\Lambda_{QCD})$ and requires for a concrete choice. It is possible, for instance, to cut out the series at the optimal number of terms: $n \simeq 2\pi/(b_o\alpha_s)$. In what follows we choose the definition of the quark pole mass which looks most natural, i.e. the principle value Borel resummation (PVBR) of the above series. This was performed in [14], so that the result for $\kappa_Q^{(m)}$ in Eq.(1) is known (in the improved leading order approximation, ILO, i.e. by replacing constant α_s by running α_s in one loop corrections).

¹In fact, the authors of [9] renounced their results on M_c and M_b and used $M_c = 1.35 - 1.40 \text{ GeV}$, $M_b = 4.8 \text{ GeV}$ in their later articles [11].

To illustrate the characteristic numbers, let us take (see below): $M_c = (1.65 \pm 0.05) GeV$, $M_b = (5.04 \pm 0.05) GeV$. Then (α_s is in the \overline{MS} -scheme: $\alpha_s(M_c) = 0.310$, $\alpha_s(M_b) = 0.204$): $\kappa_c^{(m)} = 1.75$, $\kappa_b^{(m)} = 2.10$ [14]:

$$M_c = \overline{M}_c \left[1 + \frac{4}{3} \frac{\alpha_s(M_c)}{\pi} (1 + 1.04 + \dots) \right] = \overline{M}_c \left[1 + \frac{4}{3} \frac{\alpha_s(M_c)}{\pi} 1.75 \right] = 1.23 \overline{M}_c, (2)$$

$$M_b = \overline{M}_b \left(1 + \frac{4}{3} \frac{\alpha_s(M_b)}{\pi} (1 + 0.62 + \dots) \right) = \overline{M}_b \left[1 + \frac{4}{3} \frac{\alpha_s(M_b)}{\pi} 2.10 \right] = 1.18 \overline{M}_b. (3)$$

Therefore, the value $M_c = 1.65 GeV$ found below from the data on $\Gamma(D \rightarrow e\nu + X)$ leads to: $\overline{M}_c = 1.34 GeV$, which is $\simeq 90 MeV$ above the value $\overline{M}_c = 1.25 GeV$ obtained long ago [15] from the charmonium sum rules. The value $M_b = 5.04 GeV$ obtained below from $M_c = 1.65 GeV$ and HQET mass formulae leads to: $\overline{M}_b = 4.27 GeV$, in good agreement with the value $\overline{M}_b = 4.25 GeV$ obtained in [15] from the epsilononium sum rules.

It seems natural to expect that the accuracy of \overline{M}_b obtained from the epsilononium sum rules is better in comparison with \overline{M}_c obtained from the charmonium. So, supposing the value $\overline{M}_b = 4.27 GeV$ is sufficiently accurate (say, within a several tens MeV), we can then turn around the chain of reasoning. Starting from this value we obtain $M_b = 5.04 GeV$ from Eq.(3),² and $M_c = 1.65 GeV$ from the HQET mass formulae, with each step having a several tens MeV typical accuracy.

4. $D^+ \rightarrow l\nu + X$. Determination of M_c .

On account of radiative, $O(\Lambda_{QCD}^2/M_c^2)$ and four-fermion operator ($O(\Lambda_{QCD}^3/M_c^3)$) corrections, the expression for Γ_{sl} can be represented in the form [6]:

$$\Gamma_{sl}(D^+) = \Gamma_{Born}^{sl} I_{rad} \left\{ z_o N_c \left[1 - \frac{2}{N_c} \frac{z_1}{z_o} \frac{\mu_G^2}{M_c^2} \right] + \delta_{lept}^{(c)} \pm \Delta \right\}, (4)$$

$$\Gamma_{Born}^{sl} = \frac{G_F^2 M_c^5}{192 \pi^3}, \quad N_c = \frac{\langle D | \bar{c} c | D \rangle}{2 M_D} \simeq \left(1 - \frac{\langle p^2 \rangle}{2 M_c^2} + \frac{\mu_G^2}{2 M_c^2} \right), (5)$$

$$\langle p^2 \rangle = \frac{\langle D | \bar{c} (iD)^2 c | D \rangle}{2 M_D}, \quad \mu_G^2 = \frac{\langle D | \bar{c} \frac{i}{2} g_s \sigma_{\mu\nu} G_{\mu\nu} c | D \rangle}{2 M_D}, (6)$$

² It seems, this value of M_b is stable even on account of $O(\alpha_s^2)$ corrections. On the one hand, there appeared (preliminary) results [16] on the $O(\alpha_s^2)$ -corrections to the sum rules. They are positive and enter with significant coefficients. So, they will increase slightly the above value $\overline{M}_b = 4.25 GeV$. On the other hand, the genuine $O(\alpha_s^2)$ -correction in Eq.(3) is known and is negative, so that the transition coefficient between \overline{M}_b and M_b is slightly less than 1.18. These two effects tend to compensate each other, so that the value $M_b = 5.04 GeV$ has real chances to be sufficiently precise.

$$I_{rad} \simeq \left[1 - \frac{2}{3} \frac{\alpha_s(M_c)}{\pi} f_o \kappa_c^{(w)} \right], \quad (7)$$

where z_o, z_1, f_o are known functions of m_s/M_c , $\kappa_c^{(w)}$ is the analog of $\kappa_c^{(m)}$ in Eq.(1) and $\delta_{lept}^{(c)}$ in Eq.(4) is the contribution of the four-fermion operators.

We use below ³: ⁴

$$m_s \simeq 150 \text{ MeV}; \quad \langle p^2 \rangle \simeq 0.25 \text{ GeV}^2; \quad \mu_G^2 \simeq \frac{3}{2} M_c (M_{D^*} - M_D) \simeq 0.35 \text{ GeV}^2. \quad (9)$$

The value of M_c can be determined from a comparison of Eq.(4) with the experimental value: $\Gamma_{sl}(D^+ \rightarrow l\nu + X) = (1.08 \pm 0.06) \cdot 10^{-13} \text{ GeV}$. Because the dependence of Γ_{sl} on M_c is highly nonlinear, it is more convenient to proceed in an opposite way. Namely, let us show that Eq.(4) reproduces the experimental value at $M_c \simeq 1.65 \text{ GeV}$. We have:

$$\alpha_s(M_c^2) \simeq 0.310, \quad f_o \simeq 3.25, \quad N_c \simeq 1.02, \quad \Gamma_{Born}^{sl} \simeq 2.80 \cdot 10^{-13} \text{ GeV}, \quad (10)$$

$$\Gamma_{sl}(D^+) \simeq \Gamma_{Born}^{sl} I_{rad} \{0.71 + \delta_{lept}^{(c)} \pm \Delta\}. \quad (11)$$

As for $\kappa_c^{(w)}$, the two loop radiative correction $O(b_o \alpha_s)$ was calculated in [20]:

$$I_{rad} = \left[1 - \frac{2}{3} \frac{\alpha_s(M_c)}{\pi} f_o (1 + 1.03 + \dots) \right]. \quad (12)$$

Comparing with Eq.(2), it is seen that both series follow the same pattern. Besides, because the leading renormalon is the same in both series, it seems natural that $\kappa_c^{(w)}$ will be close to $\kappa_c^{(m)}$. ⁵ Therefore, we estimate: $\kappa_c^{(w)} \simeq 1.75$, so that $I_{rad} \simeq 0.626$.

³Our definition of $\langle p^2 \rangle$ (and, analogously, $\bar{\Lambda}$ and others) differs from the cut off dependent $\langle p^2 \rangle_\mu$ used by I.Bigi et.al. If one represents this last (at $\mu \gg \Lambda_{QCD}$) as:

$$\langle p^2 \rangle_\mu = C_o \frac{\alpha_s}{\pi} \mu^2 + \dots + C_1 \Lambda_{QCD}^2 + C_2 \frac{\Lambda_{QCD}^3}{\mu} + \dots, \quad (8)$$

then our $\langle p^2 \rangle$ corresponds to the cut off independent term $C_1 \Lambda_{QCD}^2$. We prefer this definition because it selects the universal number, while all μ -dependent terms will be canceled finally in observable quantities like decay widths. On the other hand, with our definition some useful inequalities like $\langle p^2 \rangle_\mu \geq (\mu_G^2)_\mu$ will be lost, in general.

⁴From our viewpoint, the value: $\langle p^2 \rangle \simeq 0.5 - 0.6 \text{ GeV}^2$ obtained in [17] is overestimated. Let us recall [18] that the mean value of the vacuum quark 4-momentum squared is: $\langle -k_\mu^2 \rangle_o = 4/3 \langle k^2 \rangle_o \simeq 0.4 \text{ GeV}^2$, and the quarks in the pion have their momenta somewhat less on the average than the vacuum quarks [19]. Let us point out also that $\langle p^2 \rangle$ enters here to N_c only and plays no essential role.

⁵ This is supported also by the examples considered in [14].

The main contributions to $\delta_{lept}^{(c)}$ in Eq.(4) originate from the figs.1a,1b diagrams which show the matrix elements of the four-fermion operators in the effective Lagrangian [1]:

$$\left\{\delta_{lept}^{(c)}\right\}_{figs.1a,1b} \simeq \{-8.5\%\}_{fig.1a} + \{-3.5\%\}_{fig.1b} \simeq -12\% . \quad (13)$$

There are also other small contributions to $\delta_{lept}^{(c)}$, the typical one is from the four-fermion operator hidden in the Born operator $\bar{c}p^4\hat{p}c$, fig.1c ⁶:

$$\left\{\delta_{lept}^{(c)}\right\}_{fig.1c} = -\frac{2\pi\alpha_s}{M_D M_c^3} < D|\bar{c}(0)\gamma_\mu(1+\gamma_5)\frac{\lambda^a}{2}c(0)\cdot J_\mu^a(0)|D > \simeq 1\% . \quad (14)$$

Estimates show that the typical value of next corrections (denoted by $\pm\Delta$ in Eq.(4)) is a few per cent. To be safe, we take: $\Delta = 6\%$. So :

$$\Gamma_{sl}(D^+) \simeq 0.375 \Gamma_{Born}^{sl} (1 \pm 10\%) \simeq (1.05 \pm 0.10) \cdot 10^{-13} GeV , \quad (15)$$

in agreement with data. Because the decay width is highly sensitive to the precise value of M_c , ⁷ this later is tightly constrained: ⁸

$$M_c = (1.65 \pm 0.05) GeV . \quad (16)$$

We were going in this section into calculation details as the precise value of M_c is the base of all further calculations, and because the above value of M_c is essentially higher than the commonly accepted at present values $M_c = 1.35 - 1.45 GeV$.

5. Mass formulae: M_b and $\bar{\Lambda}$.

The HQET mass formulae look as ($\bar{M} = (M_P + 3M_V)/4$):

$$M_b - M_c = \bar{M}_B - \bar{M}_D + \left[\frac{\langle p^2 \rangle}{2M_c} - \frac{\langle p^2 \rangle}{2M_b} \right] + O\left(\frac{\Lambda_{QCD}^3}{M_c^2}\right) , \quad (17)$$

$$M_B = M_b + \bar{\Lambda} + \frac{\langle p^2 \rangle - \mu_G^2}{2M_b} + O\left(\frac{\Lambda_{QCD}^3}{M_b^2}\right) , \quad (18)$$

The expected accuracy of Eq.(17) is a several tens MeV, ⁹ and even better in Eq.(18). We obtain therefore (with the accuracy $\pm 50 MeV$, determined mainly by the uncertainty in M_c):

$$M_b = 5.04 GeV , \quad \bar{\Lambda} = 250 MeV . \quad (19)$$

⁶Our result here differs by the factor 1/2 from those obtained in [21]

⁷For instance, with $M_c = 1.55 GeV$ the calculated $\Gamma_{sl}(D)$ will be more than 40% smaller.

⁸Really, higher order corrections are arranged in Eq.(4) in such way to obtain a minimal possible value of M_c . Therefore, $M_c = 1.65 GeV$ is rather a lower bound.

⁹Supposing that $\langle p^2 \rangle$ is within the "reasonable" interval, say: $0.25 \pm 0.05 GeV^2$.

These results differ significantly from the widely used values: $M_b = 4.8 \text{ GeV}$, $\bar{\Lambda} = 500 \text{ MeV}$.

6. f_D and f_B .

The knowledge of precise values of the decay constants f_D and f_B is of crucial importance for many calculations with the D and B mesons (analogously to f_π for the pion). The calculated values of f_D , f_B are highly sensitive to the precise values of M_c , M_b (more precisely, to $M_D - M_c \simeq M_B - M_b \simeq \bar{\Lambda}$), and increase quickly with increasing $\bar{\Lambda}$.

The QCD sum rules for the chiral current correlator which is a difference of the pseudoscalar and scalar current correlators, posses the advantage of being protected, in the chiral limit, against pure perturbative contributions (which are poorly controllable in separate correlators), and having no significant loop corrections to non-perturbative contributions. Using them and the above values of M_c , M_b , we obtain [1]:

$$f_D(M_c) \simeq 165 \text{ MeV}, \quad f_B(M_b) \simeq 120 \text{ MeV}, \quad (20)$$

(with the expected accuracy about 10%, which is always difficult to estimate reliably when dealing with the QCD sum rules).

While the above value of f_D is only slightly below the widely accepted at present value $\simeq 180 \text{ MeV}$, the above value of f_B is much smaller than the widely used values $\simeq 180 - 200 \text{ MeV}$.¹⁰

7. Difficulties with the naive lifetime estimates.

On account of the Born term, the leading radiative and $O(\Lambda_{QCD}^2/M_c^2)$ corrections (all nonvalence), the D meson hadronic width is ($\Gamma_{Born} = G_F^2 M_c^2 / 64 \pi^3$):¹¹

$$\Gamma_{nl}^o \simeq \Gamma_{Born} [1.50]_{rad} z_o N_c [1.07]_{\mu_G} \simeq 1.54 \Gamma_{Born}. \quad (21)$$

At the level $O(\Lambda_{QCD}^3/M_c^3)$ there appear first valence (and additional nonvalence) contributions to the decay widths originating from the four-fermion operators. The effective Lagrangian (normalized at $\mu_o^2 = 0.5 \text{ GeV}^2$) has the form [1]:¹²

$$L_{eff}^c \simeq \frac{G_F^2}{2\pi} \left\{ \bar{\lambda}^2 g_{\mu\nu} L_{\mu\nu}^d + \lambda^2 t_{\mu\nu}(\lambda) L_{\mu\nu}^u + L_{PNV} + \dots \right\}, \quad (22)$$

¹⁰As for the QCD sum rule calculations, the large value of f_B originates mainly from using small values of M_b (i.e. $\bar{\Lambda} \simeq 500 - 600 \text{ MeV}$). As for the lattice calculations, the predictions for f_B decrease with time, starting from $\simeq 250 - 300 \text{ MeV}$ 1-2 years ago and reaching now $140 \pm 20 \text{ MeV}$ in the latest paper [22].

¹¹Unlike the semileptonic width, there are two μ_G^2/M_c^2 corrections, each one $\simeq 30\%$ but they cancel strongly each other [6].

¹²Only the most important terms are shown.

$$L_{\mu\nu}^d = \left\{ -1.1 (\bar{c} \Gamma_\mu d) (\bar{d} \Gamma_\nu c) + 4.0 (\bar{c} \Gamma_\mu \frac{\lambda^a}{2} d) (\bar{d} \Gamma_\nu \frac{\lambda^a}{2} c) \right\}, \quad (23)$$

$$L_{\mu\nu}^u = 3.3 (\bar{c} \Gamma_\mu \frac{\lambda^a}{2} u) (\bar{u} \Gamma_\nu \frac{\lambda^a}{2} c), \quad t_{\mu\nu}(\lambda) = \frac{1}{3} \left(\frac{\lambda_\mu \lambda_\nu}{\lambda^2} - g_{\mu\nu} \right), \quad (24)$$

where λ (or $\bar{\lambda}$) is the total 4-momentum of the integrated quark pair. It can be read off from each diagram in fig.2 and differ from P_c by the spectator quark momenta. The term L_{PNV} is nonvalence and originates from the diagram in fig.1a.

Let us try now to obtain the estimate of $\langle D^+ | L_{eff} | D^+ \rangle$ using the factorization approximation and neglecting the spectator quark momenta in comparison with M_c [4], [23]. Then, L^u and the second term in L^d give zero contributions and:

$$\Delta \Gamma_{factor}(D^+) \simeq \left[-1.1 \cdot 16 \pi^2 \frac{f_D^2(\mu_o) M_D}{M_c^3} \right] \Gamma_{Born} \simeq -1.50 \Gamma_{Born}. \quad (25)$$

The term L_{PNV} in Eq.(22) adds $\simeq -0.15 \Gamma_{Born}$ [1], so that we obtain for the hadronic width of D^+ :

$$\Gamma_{nl}(D^+) \simeq (1.54 - 1.50 - 0.15) \Gamma_{Born} \simeq -0.11 \Gamma_{Born}, \quad (26)$$

which does not make much sense. It is clear that the above approximations are too poor and some estimates are essentially wrong.

8. Non-factorizable contributions, λ and $\bar{\lambda}$.

It is seen from Eqs.(22)-(24) that (internally) coloured operators whose matrix elements are zero in the factorization approximation, enter L_{eff} with much larger coefficients. So, even if their matrix element are suppressed, they may be of importance. This is really the case. For instance, the most important weak annihilation non-factorizable contributions shown in fig.2 were estimated using the QCD sum rules [1]:

$$\langle D(p) | (\bar{c} \Gamma_\mu \frac{\lambda^a}{2} q) (\bar{q} \Gamma_\nu \frac{\lambda^a}{2} c) | D(p) \rangle_{fig.2a,b} \simeq \frac{1}{15} f_D^2 M_D^2 \left(\frac{p_\mu p_\nu}{p^2} - g_{\mu\nu} \right), \quad (27)$$

$$\langle B(p) | (\bar{b} \Gamma_\mu \frac{\lambda^a}{2} q) (\bar{q} \Gamma_\nu \frac{\lambda^a}{2} b) | B(p) \rangle_{fig.2a,b} \simeq \frac{1}{20} f_B^2 M_B^2 \left(\frac{p_\mu p_\nu}{p^2} - g_{\mu\nu} \right), \quad (28)$$

(the contribution due to the fig.3 diagram is smaller, the fig.2c contribution is negligibly small). Comparing with the factorizable contribution:

$$\langle D(p) | (\bar{c} \Gamma_\mu q) (\bar{q} \Gamma_\nu c) | D(p) \rangle \simeq f_D^2 M_D^2 \left(\frac{p_\mu p_\nu}{p^2} \right), \quad (29)$$

we see that the factorization approximation works very well, even for the D mesons.

But there is one more factor which suppresses heavily the factorizable contributions in the D meson case: the phase space of the integrated quark pair is much smaller for the crossed contributions (fig.2b) in comparison with the direct ones (fig.2a). I.e., the characteristic value of λ^2 in Eq.(22) is much larger than those of $\bar{\lambda}^2$. Really (see figs. 2a, 2b): $\lambda \simeq P_c + k = P_D$, while $\bar{\lambda} \simeq P_D - k_1 - k_2$, where k_1, k_2 are the spectator quark momenta. It can be shown that the momentum fraction carried by the spectator quark is: $\langle x \rangle \simeq 2 \left(1 - M_Q^2/M_P^2\right)/3$ (M_P is the meson mass), and is $\simeq 15\%$ for the D meson and $\simeq 6\%$ for the B one. So, the simplest estimate of $\langle \bar{\lambda}^2 \rangle$ looks as: $\langle \bar{\lambda}^2 \rangle / M_P^2 \sim (1 - 2 \langle x \rangle)^2$ and is $\simeq 0.5$ for the D meson and $\simeq 0.8$ for the B one. More accurate calculations [1] give even stronger effect for the D meson:

$$\langle \bar{\lambda}^2 \rangle_D \simeq 0.35 M_D^2, \quad \langle \bar{\lambda}^2 \rangle_B \simeq 0.80 M_B^2. \quad (30)$$

As a result of all the above effects, the non-factorizable contributions become comparable with the factorizable one.

9. $D^{\pm,0,s}$ decay widths.

Accounting for all the above described contributions, one obtains [1] (in units 10^{-13} GeV , the experimental values are given in brackets, $\Gamma_{Born} = 8.4$):

$$\Gamma_{tot}(D^+) \simeq \Gamma_{Born} [1.54 - 0.27 - 0.62 - 0.21 + (0.26)_{lept}] = 0.70 \Gamma_{Born} = 5.9 \{ 6.2 \} (31)$$

$$\Gamma_{tot}(D^0) \simeq \Gamma_{Born} [1.54 - 0.27 + 0.33 + (0.26)_{lept}] = 1.86 \Gamma_{Born} = 15.6 \{ 15.8 \} (32)$$

$$\Gamma_{tot}(D^s) \simeq \Gamma_{Born} [1.54 - 0.27 - 0.07 + (0.29)_{lept}] = 1.49 \Gamma_{Born} = 12.5 \{ 13.8 \} (33)$$

In Eqs.(31)-(33): the origin of "1.54" is explained in Eq.(21), and the term "0.27" is the nonvalence contribution from figs.1a,1b diagrams ("0.15" and "0.12" respectively). The valence terms: "-0.62" in Eq.(31) is the factorizable interference contribution (fig.2b without gluons); "-0.21" in Eq.(31) is the summary non-factorizable contribution of fig.2b and fig.3 diagrams; "0.33" in Eq.(32) is the non-factorizable direct annihilation contribution, fig.2a; the small term "-0.07" in Eq.(33) originates from the Cabibbo-suppressed contributions and the non-factorizable annihilation contributions.

The expected accuracy of $\Gamma_{tot}(D_i)$ calculations is not high: $\simeq 20\%$.¹³ It is sufficient however to keep all the main contributions well under control, and is much better in comparison with all previous estimates which were qualitative rather than quantitative (see sect.7).

¹³Besides, the SU(3) symmetry breaking corrections are not accounted for in calculations with D_s .

10. $B^{\pm,o,s}$ lifetimes and $B^o - \bar{B}^o$ mixing.

The $(\bar{u}d)$ — part of the B^o width can be represented in the form:

$$\Gamma_{(ud)}(B^o) \simeq \Gamma_{Born} \frac{(2C_+^2 + C_-^2)}{3} z_o^{ud} I_{rad}^{(ud)} \Omega_{power}, \quad (34)$$

where: $(2C_+^2 + C_-^2)/3 \simeq 1.127$ is the renormalization factor due to the evolution from M_W to M_B [24], $\Omega_{power} \simeq 0.99$ is the summary effect of all power corrections [1], $z_o^{ud} \simeq 0.46$ is the phase space factor, and $I_{rad}^{(ud)} \simeq 1.03$ [25] describes all other $O(\alpha_s)$ corrections. For the $(\bar{c}s)$ part: $z_o^{cs} \simeq 0.13$, $I_{rad}^{(cs)} \simeq 1.3$ [26], [25]. On the whole (see sect.11 for the semileptonic decays, $\Gamma_{Born} = G_F^2 M_b^5 |V_{cb}|^2 / 64\pi^3 \simeq 3.93 \cdot 10^{-13} \text{ GeV}$, $\tau_{Born} \simeq 1.67 \text{ ps}$, the data see in [27]):

$$\begin{aligned} \Gamma_{tot}(B^o) &\simeq \Gamma_{Born} \left[0.53_{(ud)} + 0.19_{(cs)} + 2 \cdot 0.114_{(\epsilon\nu+\mu\nu)} + 0.028_{(\tau\nu)} + 0.02_{(b \rightarrow u)} \right] \simeq \\ &\simeq 1.00 \cdot \Gamma_{Born} \simeq 3.93 \cdot 10^{-13} \text{ GeV}, \quad \Gamma_{tot}^{exper}(B^o) = (4.10 \pm 0.18) \cdot 10^{-13} \text{ GeV}, \end{aligned} \quad (35)$$

$$Br_{(\epsilon\nu)} \simeq 11.4\%, \quad Br_{(\tau\nu)} \simeq 2.8\%, \quad Br_{(cs)} \simeq 19\%. \quad (36)$$

All these results agree with data ¹⁴ [27], except for $Br_{(cs)}$ which looks somewhat large in comparison with $Br_{(cs)}^{exper} = 0.04 \pm 0.07$. ¹⁵ On the other hand, the experimental value looks definitely somewhat small here.

As for the lifetime differences, the qualitative picture remains the same as for the D mesons but, of course, all the effects are much smaller. Let us denote by Γ_o the common width of all $B^{\pm,o,s}$ mesons which they have on neglect of the four-fermion operator contributions (and SU(3) breaking). Then the valence contributions look as:

$$\frac{\Delta\Gamma(B^+)}{\Gamma_o} \simeq (-2.1\%) + (-1.3\%) \simeq -3.4\%, \quad (37)$$

where (-2.1%) is the factorizable interference contribution (fig.2a without gluons), and (-1.3%) is the summary non-factorizable contribution of fig.2b and fig.3 diagrams.

The B^o and B_s mesons (neglecting SU(3) breaking) receive only the non-factorizable contributions from fig.2a and fig.3 diagrams:

$$\frac{\Delta\Gamma(B^o)}{\Gamma_o} \simeq 0.6\%, \quad \frac{\Delta\Gamma(\bar{B}_s)}{\Gamma_o} \simeq 0.5\%. \quad (38)$$

¹⁴ One can expect also that higher loop corrections will increase slightly the hadronic width.

¹⁵ Let us emphasize that $Br_{(cs)}$ will be essentially larger (≥ 0.30) for the quark masses $M_c \simeq 1.4 \text{ GeV}$, $M_b \simeq 4.8 \text{ GeV}$, and this will constitute a real difficulty.

Finally, the four-fermion operators give also the non-valence contributions, both factorizable: $\simeq (-0.6\%)$ from fig.1a diagram, and non-factorizable: $\simeq (-0.1\%)$ from fig.1b one.

On the whole, the lifetime difference of the B^o and B^\pm mesons is [1]:

$$\frac{\Gamma(B^o) - \Gamma(B^\pm)}{\bar{\Gamma}(B)} \simeq 4\%, \quad (39)$$

while those of B^o and \bar{B}_s is zero within the available accuracy.

The above described non-factorizable contributions, figs.2,3, determine also the corrections to the factorization approximation for the $\bar{B} - B$ -mixing, and appear to be surprisingly large here [1]: $\simeq -18\%$. As a result, one obtains for the B_s -mesons (neglecting SU(3)-breaking):

$$\frac{\Gamma(B_s^{short}) - \Gamma(B_s^{long})}{\bar{\Gamma}(B_s)} \simeq 6\%, \quad (40)$$

and for the "bag factor" of the $\bar{B}^o - B^o$ mass mixing:

$$< \bar{B}^o | (\bar{b} \Gamma_\nu d) (\bar{b} \Gamma_\nu d) | B^o >_{\mu=M_b} \equiv \frac{8}{3} f_B^2(M_b) M_B^2 B_B(M_b),$$

$$B_B(M_b) \simeq (1 - 18\%) = 0.82. \quad (41)$$

11. $B \rightarrow e \nu + X$: determination of $|V_{cb}|$.

The calculation proceeds in analogy with those in sect.4. The total effect of $O(\Lambda_{QCD}^2/M_b^2)$ power corrections is much smaller now ($\simeq -4\%$), and $\delta_{lept}^{(b)}$ is negligibly small ($\simeq -2 \cdot 10^{-3}$) [1]. Most important are radiative corrections which look as [20] ($f_o \simeq 2.46$ here):

$$I_{rad} \simeq \left[1 - \frac{2}{3} \frac{\alpha_s(M_b)}{\pi} f_o - 1.68 b_o \left(\frac{\alpha_s(M_b)}{\pi} \right)^2 - \dots \right] \simeq \quad (42)$$

$$\left[1 - \frac{2}{3} \frac{\alpha_s(M_b)}{\pi} f_o (1 + 0.6 + \dots) \right] = \left[1 - \frac{2}{3} \frac{\alpha_s(M_b)}{\pi} f_o \kappa_b^{(w)} \right]. \quad (43)$$

Comparing with Eq.(3) it is seen that (analogously to the charm case) two series follow the same pattern. So, we estimate (see also [14]): $\kappa_b^{(w)} \simeq 2.1$, $I_{rad} \simeq 0.776$. Therefore ($z_o^{e\nu} \simeq 0.46$):

$$\Gamma(B \rightarrow e \nu + X) \simeq 0.44 \Gamma_{Born}^{sl} I_{rad} \simeq 4.5 \cdot 10^{-14} GeV \left| \frac{V_{cb}}{0.042} \right|^2, \quad (44)$$

$$\Gamma_{Born}^{sl} = \frac{G_F^2 M_b^5 |V_{cb}|^2}{192 \pi^3} \simeq 13.1 \cdot 10^{-14} GeV \left| \frac{V_{cb}}{0.042} \right|^2. \quad (45)$$

As for the data, we take [27]:

$$\tau_B = (1.60 \pm 0.07) ps, \quad Br(B \rightarrow e\nu + X) = (11.0 \pm 0.5)\%, \quad (46)$$

and obtain:

$$|V_{cb}| = (42 \pm 1) \cdot 10^{-3} \left[\frac{Br(B \rightarrow e\nu + X)}{11.0\%} \right]^{1/2} \left[\frac{1.6 ps}{\tau_B} \right]^{1/2}. \quad (47)$$

The error bars in Eq.(47) were estimated by varying: $1.60 \leq M_c \leq 1.70 GeV$, $3.34 \leq (M_b - M_c) \leq 3.39 GeV$.¹⁶

For the $B \rightarrow \tau\nu + X$ decays: $f_o \simeq 2.0$, $z_o^{\tau\nu} \simeq 0.105$, $I_{rad} \simeq 0.83$, and so:

$$Br\left(\frac{\tau}{e}\right) \simeq 0.25. \quad (48)$$

12. The unitarity triangle.

1). Using: $V_{cb} = A \lambda^2$, $\lambda \simeq 0.221$ and (see Eq.(47)) $V_{cb} \simeq 4.2 \cdot 10^{-2}$, one has:

$$A \simeq 0.86. \quad (49)$$

2). The $B^o - \bar{B}^o$ mass difference is given by the well known formula (see i.e. [28]), and using (see above): $f_B(M_b) = 120 MeV$, $B_B(M_b) = 0.82$, and [27]

$$M_t^{pole} = 175 GeV, \quad \tau(B^o) = 1.6 ps, \quad (50)$$

one obtains :

$$x_d = \frac{\Delta M_d}{\Gamma(B^o)} = (0.78 \pm 0.06) \simeq 4.5 \cdot 10^3 |V_{td}|^2, \quad (51)$$

$$|V_{td}| = |V_{cb}| \lambda \left[(1 - \rho)^2 + \eta^2 \right]^{1/2} \simeq 1.3 \cdot 10^{-2}, \quad (52)$$

$$\left[(1 - \rho)^2 + \eta^2 \right] \simeq 2.0. \quad (53)$$

¹⁶For comparison, using $M_c = 1.4 GeV$, $M_b = 4.8 GeV$ and proceeding in the same way one obtains 44.5 instead of 42.0 in Eq.(47), and it seems it will be difficult to reconcile this value with the data on $\Gamma(B \rightarrow D^* e\nu)$.

3). Using the above given parameters and $\hat{B}_K \simeq 0.82$ from the lattice calculations, the CP-violating part of the $K^o - \bar{K}^o$ mixing can be written in the form (see i.e. [28]):

$$e^{-i\pi/4} \epsilon_K \cdot 10^3 \simeq 8.0 \hat{B}_K \eta (1.36 - \rho) = 2.26,$$

$$\eta (1.36 - \rho) \simeq 0.35. \quad (54)$$

Therefore, we obtain from Eqs.(53),(54):

$$\rho \simeq -0.40, \quad \eta \simeq 0.20, \quad \delta = \arctg\left(\frac{\eta}{\rho}\right) \simeq 0.85 \pi, \quad \left|\frac{V_{ub}}{V_{cb}}\right| \simeq 0.10, \quad (55)$$

$$\sin 2\alpha \simeq 0.60, \quad \sin 2\beta \simeq 0.28, \quad \sin 2\gamma \simeq -0.80. \quad (56)$$

The unitarity triangle is shown in fig.4. With the above parameters the CP-violating asymmetry in $B_d^o \rightarrow \Psi K_S$ decay is:

$$|A(B_d^o \rightarrow \Psi K_S)| \simeq \frac{x_d}{1 + x_d^2} \sin 2\beta \simeq 0.14. \quad (57)$$

13. Summary

It is seen from all the above presented calculations that a self-consistent picture emerges which agrees with a large number of various experimental data, and allows to obtain a number of important predictions which can be checked in future experiments.

One of our main concerns was to calculate reliably the four-fermion operator contributions. These are of crucial importance for explaining the pattern of the $D^{\pm,o,s}$ lifetimes, the $B^{\pm,o,s}$ lifetime differences and the $\bar{B} - B$ mixing. There is a clear reason explaining the importance of these four-fermion operator contributions, although they are formally only $O(\Lambda_{QCD}^3/M_Q^3)$ corrections: they are the first who gain the large numerical factor $\simeq 16\pi^2$ (see Eq.(25)) due to the two-particle phase space, in comparison with the three-particle one for the Born term and (non-valence) $O(\Lambda_{QCD}^2/M_Q^2)$ corrections. It is clear that this enhancement factor operates one time only, so that all other $O(\Lambda_{QCD}^3/M_Q^3)$ and higher order corrections are naturally small (see Eq.(14)) and have no much chances to be of real importance. ¹⁷

¹⁷ The applicability of the standard operator expansions to the calculation of $\Gamma(D \rightarrow e\nu + X)$ has been questioned in [21] on the only ground that the authors don't believe the c-quark pole mass may be as large as $M_c \simeq 1.65 GeV$. They insist it can not exceed $\simeq 1.4 GeV$. Let us emphasize that the calculated value of $\Gamma(D \rightarrow e\nu + X)$ (see Eq.(15)) will decrease $\simeq 3$ times for $M_c = 1.4 GeV$. So, there should be huge additional contributions which dominate the semileptonic width and remain invisible within the standard operator expansion. No one reliable argument is presented however in [21] to support $M_c \simeq 1.4 GeV$, and no one missed contribution is shown which (even potentially) may be of great importance.

The calculated values of the c and b quark pole masses, $M_c \simeq 1.65 \text{ GeV}$, $M_b \simeq 5.04 \text{ GeV}$, appeared to be significantly larger the widely accepted at present values: $M_c \simeq 1.35 - 1.45 \text{ GeV}$, $M_b \simeq 4.8 \text{ GeV}$. This difference is of great importance, as most of calculations with heavy quarks are highly sensitive to precise values of their masses. In particular, the calculated decay constant $f_B \simeq 120 \text{ MeV}$ appeared to be much smaller the widely accepted at present value $f_B \simeq 180 - 200 \text{ MeV}$. This difference leads, in its turn, to essentially different predictions for the $B^0 - B^\pm$ lifetime difference, $\overline{B}^0 - B^0$ mixing and the unitarity triangle parameters. Just because the above obtained values of M_c , M_b , $\overline{\Lambda}$, f_B , etc, look highly non-standard at present, we described above the calculations of many experimentally measured quantities to show there is no disagreement with data. Besides, a number of concrete predictions is described which can be checked in future experiments (see Tables).

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Table 1: **INPUT**

$\Gamma(D \rightarrow e\nu + X)$	$(1.08 \pm 0.06) \cdot 10^{-13} \text{ GeV}$
$\tau(B^0)$	$(1.60 \pm 0.07) \text{ ps}$
$Br(B \rightarrow e\nu + X)$	$(11.0 \pm 0.5)\%$
$x_d = \Delta m_d / \Gamma_{B^0}$	0.78 ± 0.06
ϵ_K	$2.26 \cdot 10^{-3}$
\tilde{B}_K	0.82 ± 0.05
M_t^{pole}	175 GeV
$\lambda = V_{us} $	0.221
$\alpha_s^{\overline{MS}}(M_W)$	0.118 MeV
$[f_B^2 B_B(M_b)]_{B_s} / [f_B^2 B_B(M_b)]_{B_d}$	1.3

Table 2: **OUTPUT**

M_c^{pole}	1.65 GeV
M_b^{pole}	5.04 GeV
$f_D(M_c)$	165 MeV
$f_B(M_b)$	120 MeV
$B_B(M_b)$	0.82
$[\Gamma(B^0) - \Gamma(B^\pm)] / \Gamma_B$	4%
$[\Gamma(B_s^{short}) - \Gamma(B_s^{long})] / \overline{\Gamma}(B_s)$	6%
$\Delta m_s / \Delta m_d$	14
$ V_{cb} $	$4.2 \cdot 10^{-2}$
$ V_{td} $	$1.3 \cdot 10^{-2}$
$ V_{ub}/V_{cb} $	0.10
$ A(B \rightarrow \Psi K_s) $	0.14
$ \epsilon'/\epsilon \cdot 10^4$	a few units

Figure captions

Fig.1 Non-valence factorizable (a,c) and non-factorizable (b) contributions to matrix elements.

Fig.2 Valence direct (a) and cross (b) annihilation non-factorizable contributions to matrix elements of coloured operators; (c) the same for colourless operators.

Fig.3 Valence non-factorizable contribution (plus the mirror diagram).

Fig.4 The unitarity triangle: $\rho \simeq -0.40$, $\eta \simeq 0.20$.